

The JETS Challenge

Provided by Dave Meredith, Associate Professor,
Penn State University-Fayette

Challenge 86 – The Sputnik Challenge

Problem:

The space race was launched 51 years ago on October 4, 1957, when the Soviet Union successfully launched Sputnik I as the first manmade satellite to orbit the earth every 96.2 minutes. The satellite was a 58.0 cm-diameter aluminum sphere that weighed about 83 kg. The equation for orbital mechanics is given as

$$T = \frac{2\pi}{\sqrt{\mu}} r^{\frac{3}{2}}$$

where T = to complete one orbit in seconds
 r = radius of circular orbit in kilometers
 $\mu = M \bullet g$ in kg-km/s² (= 398,600 for the earth)

Find the radius of Sputnik's orbit in km.

Solution:

96.2 minutes
58 cm diameter
83 kg

$$T = \frac{2\pi}{\sqrt{\mu}} r^{\frac{3}{2}}$$

$$T = 96.2 \text{ minutes} \times 60 \text{ sec./1 min.} = 5,772 \text{ sec.}$$

$$\mu = 398,600$$

$$5,772 = \frac{2\pi}{\sqrt{398,600}} r^{3/2}$$

$$579,982.7826 = r^{3/2}$$

$$(579,982.7826)^{2/3} = r$$

$$6,954.67331 \text{ km} = r$$

$$6,955 \text{ km} = r$$