

The JETS Challenge

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Challenge 45 – The Pittsburgh Steelers Logo Challenge

Problem:

The logo of the Superbowl champion Pittsburgh Steelers has a mathematical legacy that goes back to Sir Isaac Newton and Sir Christopher Wren. Mathematically the logo consists of three interlocked hypocycloids called astroids. A hypocycloid with four cusps is created by rolling a circle of radius R inside a circle of radius $4R$. The formula for this shape is given by $x^{2/3} + y^{2/3} = R^{2/3}$ and the area contained within each astroid is $3/2$ times the area of the rolling circle.

What fraction of the area of the circular logo (outlined in the dotted line) is contained within the three asteroids together?



Solution:

$$\begin{aligned}\text{Area Astroid} &= \frac{3}{2} \text{ rolling circle} \\ &= \frac{3}{2} \pi R^2\end{aligned}$$

$$3 \cdot \text{Astroids} = 3 \cdot \frac{3}{2} \pi R^2$$

$$A(\text{big circle}) = \pi(8R)^2$$

$$\% = \frac{\frac{9}{2} \pi R^2}{64 \pi R^2} = \frac{9}{2} \cdot \frac{1}{64} = \frac{9}{128} = .0703125$$

$$= 7.03125\%$$